Calculus AB

2-2

Derivatives

Find the derivative:

$f(x) = x^4$

$f'(x) = 4x^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$F'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} = \frac{(x+\Delta x)^4 - x^4}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{(\cancel{1x^4} + 4x^3\Delta x + 6x^2\Delta x^2 + 4x\Delta x^3 + 1\Delta x^4) - \cancel{x^4}}{\Delta x} = \frac{4x^3\Delta x + 6x^2\Delta x^2 + 4x\Delta x^3 + \Delta x^4}{\Delta x}$$

$$\lim_{\Delta x \to 0} 4x^3 + 6x^2\Delta x + 4x\Delta x^2 + \Delta x^3 = \boxed{4x^3}$$

Pascal's Triangle

Derivative

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

This row is circled because they all have one Δx. This Δx gets canceled by the one in the denominator. The circled numbers represent 1, $2x$, $3x^2$, $4x^3$, $5x^4$, etc, which is the result of the power formula on the next slide. All the numbers below the circled row will still have Δx in which the limits will approach zero, thus the product will remove these terms from the derivative. This is why the power rule works!

The Power Rule -

If n is a rational number, then $f(x) = x^n$ is differentiable and

derivative of

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

with respect to

Find the derivative of each function. (pg 113)

6) $y = x^{16}$

$y' = 16x^{15}$

or

$\frac{dy}{dx} = 16x^{15}$

*) $y = 8$

$\frac{dy}{dx} = 0$

The Constant Rule -

$$\frac{d}{dx}\left[cf(x)\right] = cf'(x)$$

*) $f(x) = 3x^2$

$F'(x) = 6x$

The Sum and Difference Rules -

$$\frac{d}{dx}\left[f(x) + g(x)\right] = f'(x) + g'(x)$$

$$\frac{d}{dx}\left[f(x) - g(x)\right] = f'(x) - g'(x)$$

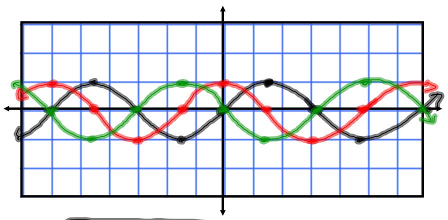
14) $f(t) = t^2 + 2t - 3t^0$

$F'(t) = 2t + 2t^0 + 0$

$t^0 = 1$, thus $2t^0 = 2$

$f(x) = \sin(x)$

Graph $f'(x)$:



$f'(x) = \cos x$

$f''(x) = -\sin x$

$\boxed{\frac{d}{dx} \sin x = \cos x}$

$\boxed{\frac{d}{dx} \cos x = -\sin x}$

Derivatives of the Sine and Cosine Functions

$f(x) = \sin(x)$, $f'(x) =$ $\underline{\cos x}$

$g(x) = \cos(x)$, $g'(x) =$ $\underline{-\sin x}$

19) $g(t) = \pi \cos t$

$g(t) = -\pi \sin t$

Complete the table.

	Original Function	Rewrite	Differentiate	Simplify
26)	$y = \frac{6}{(5x)^3}$ $= \frac{6}{125x^3}$	$= \frac{6}{125}x^{-3}$	$= \frac{-18}{125}x^{-4}$	$= \frac{-18}{125x^4}$

Find the slope of the graph of the function at the indicated point.

32) $g(t) = 3 - \frac{3}{5t}$ at $(\frac{3}{5}, 2)$ $\quad 3 - \frac{3}{5}t^{-1}$

$g'(t) = \frac{3}{5}t^{-2} = \frac{3}{5t^2}$

$m = g'(\frac{3}{5}) = \frac{3}{5(\frac{3}{5})^2} = \frac{3}{\frac{9}{5}} = \frac{15}{9} = \boxed{\frac{5}{3}}$

Find the derivative of the function.

42) $f(x) = x + \frac{1}{x^2}$ $\quad x^{-2} \to -2x^{-3}$

$f'(x) = 1 - \frac{2}{x^3}$

45) $f(x) = \frac{x^3 - 6}{x^2}$

$= \frac{x^3 - 6}{x^2} = \frac{x^3}{x^2} - \frac{6}{x^2}$

$= x - 6x^{-2}$

$f'(x) = 1 + 12x^{-3}$

$= \boxed{1 + \frac{12}{x^3}}$

$\frac{x^3 + 12}{x^3}$

Find the derivative of the function.

50) $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$ $= x^{\frac{1}{3}} + x^{\frac{1}{5}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{5}x^{-\frac{4}{5}}$

$= \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{5\sqrt[5]{x^4}}$

Assignment:

Pg. 113
1-53 odd